# Section 6: Credit Risk

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a. Define and evaluate credit risk as related to fixed income securities and derivatives counter parties
b. Define and evaluate spread risk as related to fixed income securities and derivatives.
c. Describe, contrast and assess credit risk measurement techniques and models.
d. Calculate effective duration and effective key-rate durations of a portfolio
e. Contrast modified duration and effective duration measures
f. Explain the concepts of immunization including modern refinements and practical limitations

# Risk Management by Crouhy
# Chapter 8: The Credit Migration Approach to Measuring Credit Risk

## 1. Introduction

Problems with the BIS (Bank for International Settlements) requiring the capture of credit risk as the sum of concentration risk, spread risk, downgrade risk, and default risk

1. Spread risk is related to both market risk and credit risk; downgrade risk is only related to credit spread risk

   If the two are added together there could be double counting

2. The market anticipates future credit events and adjusts the spreads accordingly
3. Default risk is simply a type of downgrade risk
4. Market and economic conditions can affect the credit rating, so the market risk and credit risk should be fully integrated

New approaches to credit modeling

1. CreditMetrics

   Analysis of credit migration (covered later in this chapter)

   Could allow the default probabilities to vary with the credit cycle which is affected by the economic cycle (done in CreditPortfolioView at the end of this chapter)

2. Expected default frequency

   Developed by KMV corporation (covered in chapter 9)

3. CreditRisk+

   Focuses on default rather than credit migration (covered in chapter 10)

4. Reduced form approach (also covered in chapter 10)

All of the methods assume deterministic interest rates, so not valid for derivatives

Need an integrated model where defaults and migration probabilities are affected by the stochastically generated economic condition

## 2. CreditMetrics Framework

Estimated forward distribution of portfolio value changes due to credit quality changes

Credit VaR challenges

1. Portfolio distribution is not normal

   Highly skewed to the left because credit upgrades have limited upside while downgrades have big downsides

2. Diversification effect is more complex

   Must consider the correlations for all pairs of obligors

3. Loan information is not complete

CreditMetrics makes no provision for market risk

## 3. Credit VaR for a Bond (Building Block 1)

<u>Step 1: Specify the transition matrix</u>

Could use Moody's, S&P's, or internal ratings

Create a table of probabilities of transitioning from one rating to another over a specified time period (usually one year)

Rating agencies publish transition matrices based on averages

  Banks should use with care because may not be too similar to loans and bonds in their portfolio

Moody and S&P also produce cumulative default rates

  Assuming a stationary Markovian process, multiplying the one-year transition matrix *n* times will generate the *n*-year cumulative default rate matrix

The transition matrix should be adjusted to account for the current economic environment

<u>Step 2: Specify the credit risk horizon</u>

Must be consistent with the transition matrix from Step 1

Usually set at one year because fits well with the available accounting data and reports published by rating agencies

Step 3: Specify the forward pricing model

A forward spot curve must be generated for each credit rating (AAA, AA, …)

The remaining future cash flows are discounted using the applicable spot curve to determine the forward price of the security at each rating

Spreads tend to decrease over time for low-grade bonds and increase over time for high-grade bonds

If the issuer defaults, a specified recovery rate of the par value is returned to the investor (usually 17% to 54%)

A beta distribution for the default recovery rate is assumed for simulations

Step 4: Derive the forward distribution of the changes in portfolio value

Using the probabilities from the transition matrix, one can determine the distribution of the changes in portfolio value

As mentioned earlier the distribution will be skewed to the left

**4. Credit VaR for a Loan or Bond Portfolio (Building Block 2)**

A joint transition matrix is easy to determine for two bonds if one assumes there is no correlation between changes in credit quality

Unfortunately this is not a reasonable assumption

An integral portion of this process is estimating the correlations because the overall credit VaR is very sensitive to the correlations

Default correlations are higher for firms in the same industry

The correlations are also higher in bad economic environments, which means the correlations will not be stationary

CreditMetrics estimates the correlation between the stock prices of various obligors to determine the credit quality change correlation

Theoretical framework developed by Merton

$V_t = B_t(F) + S_t$  *where*: $V_t$ = Total Asset Value

$B_t$ = Percentage Market Value of Bond at Time t

$F$ = Face Value of Zero-Coupon Bond

$S_t$ = Equity Value at Time t

Defaults will occur at maturity (time "T") if $V_T$ is less than F

First create a distribution of the ending asset values ($V_T$), as illustrated in Figure 8.7 on page 334

The shaded portion represents the probability of default because $V_T$ is less than F

Then divide the $V_T$ normal distribution into regions that correspond to the transition matrix probabilities

Can solve Z-values that correspond to the probabilities using a cumulative normal distribution table

For example, $N(Z_{BBB}) = 0.8053 + 0.0884 + 0.0100 + 0.0106 = 0.9143$

$\therefore Z_{BBB} = 1.37$

The probability of defaulting $N(Z_{CCC})$

$Z_{CCC}$ is the default threshold

Based on the option pricing model, $Z_{CCC} = -d_2$ because $N(-d_2)$ is the probability of default

$$d_2 = \frac{\ln\left(\frac{V_0}{V_{Def}}\right) + \left(\mu - \frac{\sigma^2}{2}\right)(t)}{\sigma\sqrt{t}}$$

This is the Black-Scholes equation with μ substituted for *r*

Using the above equation one could solve for $V_{Def}$ to determine the asset value which would trigger a default

Also, if $Z_B$ is set equal to $-d_2$, $V_{Def}$ would represent the asset value below which the issuer would be in default or rated CCC

A joint rating probability table can be derived assuming a bivariate normal distribution with some correlation ρ

Can use this table to determine probabilities such as the first company being in default and the second company having an A rating

## 5. Analysis of Credit Diversification (Building Block 2, Continuation)

Monte Carlo simulation must be used if there are more than two obligors

Use the following steps:

1. Derive the asset return thresholds for each rating
2. Estimate the correlation between each pair of obligor's asset returns
3. Generate asset return scenarios from a multivariate normal distribution with the estimated correlation matrix
4. Map the asset return for each obligor for each scenario into the corresponding rating
5. Revalue the portfolio using the applicable spread curve
6. Repeat many times (~100,000) and plot probability distribution of value changes

## 6. Credit VaR and Calculation of the Capital Charge

The economic capital is used to absorb unexpected losses

Capital = EV – P(c), where

P(c) is the value of the portfolio in the worst case scenario at the (1−c)% confidence level

EV is the expected value of the portfolio

## 7. CreditMetrics as a Loan/Bond Portfolio Management Tool: Marginal Risk Measures (Building Block 2, Continuation)

CreditMetrics calculates the impact of each individual asset on the overall portfolio standard deviation

This allows one to reduce risk effectively through trades

## 8. Estimation of Asset Correlations (Building Block 3)

Default return correlations are derived from asset return correlations, which are proxied by equity returns

The calculations are reduced by mapping obligors into countries and industries

**9. Exposures (Building Block 4)**

This is the forward pricing model applied to each credit rating

For many derivatives (like swaps and forwards) the future exposure is affected by future interest rate levels

CreditMetrics assumes an average exposure for these types of derivatives, which is clearly not an ideal approach

**10. Conditional Transition Probabilities: CreditPortfolioView**

CreditPortfolioView links default and migration probabilities to macroeconomic factors

Industries will react differently to various macroeconomic factors

Non-investment-grade defaults and credit downgrades are more common during recessions

A transition matrix is used to model the state of the economy

**Appendix 1: Elements of Merton's Model**

*Merton's model is covered sufficiently in Chapter 9, which is also on the syllabus*

**Appendix 2: Default Prediction – The Econometric Model**

Default probabilities are modeled as a logit function as follows:

$$\text{Prob}_{j,t} = \frac{1}{1+e^{-Y_{j,t}}}$$

The macroeconomic index, $Y_{j,t}$, is defined as follows

$$Y_{j,t} = \beta_{j,0} + \beta_{j,1}X_{j,1,t} + \beta_{j,2}X_{j,2,t} + \cdots + \beta_{j,m}X_{j,m,t} + v_{j,t}$$

The probability can be defined for each country / industry

**Appendix 3: Transition Matrix over a Period of less than One Year**

$$T^{\frac{1}{n}} = X^{-1}\Lambda^{\frac{1}{n}}X$$

# Risk Management by Crouhy
# Chapter 9: The Contingent Claim Approach to Measuring Credit Risk

## 1. Introduction

Major weakness of CreditMetrics

Reliance on historical averages to determine rating transition probabilities

Two critical assumptions of CreditMetrics

1. Firms in same rating class have same default rate and spread curve (even when recovery rates are different)

   But default rates evolve continuously and ratings are only adjusted discretely

2. Actual default rate equals the historical average default rate

The distribution of average default rates varies greatly based on the assumed asset correlation (i.e. how much the obligors are affected by general economic conditions)

As can be seen in Figure 9.1 on page 358, the distribution is skewed to the right (the median is greater than the mean)

Using a structural approach (rather than a credit migration approach), each case can be analyzed individually

This can develop a more precise analysis but also requires more input information

Ways to implement structural approach

1. Option pricing model approach (Merton)

   The firm's liabilities (debts) are a contingent claim on the assets because the shareholders have a right to default (and will do so if the assets are worth less than the liabilities)

   Loss rate based on firm's asset value, volatility, and risk-free rate

2. Option pricing model approach (Longstaff and Schwartz)

   Bankruptcy occurs when a firms assets drop below a specified level

   Does not account for priority structure of debt

3. Reduced form approach

   Does not explicitly depend on firm's asset value

   Covered more in chapter 10

## 2. A Structural Model of Default Risk: Merton's (1974) Model

Assume a firm has risky assets V financed by equity S and one zero-coupon debt maturing at F and market value B

The firm will default if $V_T < F$ (then ending assets are worth less than the ending liability value)

At time 0, $B_0 = Fe^{-y_T t} < Fe^{-rt}$ because there is default risk on the bond (so it must earn more than the risk-free rate "r")

The default spread risk premium is $\pi_T = y_T - r$

Assuming a frictionless market, $V_0 = S_0 + B_0$

Credit risk is a function of

1. Leverage Ratio = $LR = \frac{Fe^{-rT}}{V_0}$
2. Volatility of return on firm's assets (σ)
3. Time to maturity of debt (T)

A bank could eliminate the credit risk by purchasing a put option on V with a strike price of F

| | Time 0 | Time T | |
|---|---|---|---|
| Value of assets | $V_0$ | $V_T \le F$ | $V_T > F$ |
| | | | |
| Bank makes a loan | $-B_0$ | $V_T$ | F |
| Bank buys a put | $-P_0$ | $F - V_T$ | 0 |
| Loan plus put | $-B_0 - P_0$ | F | F |

With the purchase of the put, the bank has effectively invested $B_0 + P_0$ that will grow to a value of F even if a default occurs ($V_T \le F$)

The value of the put can be calculated from the Black-Scholes model

$$P_0 = Fe^{-rT}N(-d_2) - V_0 N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{V_0}{Fe^{-rT}}\right) + (\frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

This illustrates the cost of credit risk is a function of the firm's asset volatility, time, and the risk-free rate

Yield to maturity for corporate debt ($y_T$)

$B_0 = Fe^{-yT} \qquad P_0 = Fe^{-rT} - B_0$

The value of the put equals the expected value of the defaults, which is the difference between the present value at the risk-free rate (r) and the present value at the risk-adjusted rate (y)

$$y_T = -\left(\frac{1}{T}\right)\ln\frac{B_0}{F} = -\left(\frac{1}{T}\right)\ln\left(\frac{Fe^{-rT} - P_0}{F}\right)$$

$$= -\left(\frac{1}{T}\right)\ln\left(\frac{Fe^{-rT} - Fe^{-rT}N(-d_2) + V_0N(-d_1)}{F}\right) = -\left(\frac{1}{T}\right)\ln\left(e^{-rT}N(d_2) + \frac{V_0N(-d_1)}{F}\right)$$

$$= r - \left(\frac{1}{T}\right)\ln\left(N(d_2) + \frac{V_0}{Fe^{-rT}}N(-d_1)\right)$$

The risk-adjusted yield on the bond is a function of the risk-free rate, the leverage ratio, and the asset volatility

**3. Probability of Default, Conditional Expected Recovery Value, and Default Spread**

The Black-Scholes equation for a put can be rearranged as follows

$$P_0 = \left[-\frac{N(-d_1)}{N(-d_2)}V_0 + Fe^{-rT}\right]N(-d_2)$$

The above equation can be broken into three factors:

1. $\frac{N(-d_1)}{N(-d_2)}V$ is the expected recovery value on the bond given default
2. $Fe^{-rT}$ is the present value of a riskless bond
3. $N(-d_2)$ is the probability of default

The formulas shown above are for a risk-neutral world

In the real world (i.e. not risk-neutral) the formulas are the same except substitute μ for r in the equations for $d_1$ and $d_2$

**4. Estimating Credit Risk as a Function of Equity Value**

It is often complex to observe a firm's assets (V), so the above formulas are more difficult to apply

The assets could be treated as the sum of the equity and debt, but the corporate loans are often not traded securities

The equity for publicly traded companies is readily available, and can be thought of as a call option on the firm's assets (the strike price being the face value of the debt)

**5. KMV Approach**

Derives the default probability for each obligor based on the Merton model

The default probability is a function of the firm's capital structure, asset return volatility, and current asset value

There are three stages to derive the probabilities

1. Estimate the asset value and the volatility of the asset return

   Assumes the percentage change in asset returns is normally distributed and the volatility remains relatively constant over time

   Only the equity is directly observable, so it must be used to estimate the asset return volatility

2. Calculate the distance to default

   Bankruptcy describes a situation in which the firm's assets are being liquidated

   Default occurs when a firm misses a payment

   STD = Short-term debt LTD = Long-term debt

   DPT = Default point = STD + ½ LTD

   DD = Distance to default = Distance between $E(V_1)$ and DPT

   DD is often expressed in terms of standard deviations of asset returns

$$DD = \frac{\ln \frac{V_0}{DPT_T} + \left( \mu - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}$$

   The probability of default is N(-DD)

In some places the book (page 373, Examples 1 and 2 on pages 374-375) uses a slightly different definition for DD that is based on dollar amounts rather than percentage returns; you should be prepared for both formulas on the exam, depending upon which information is given

$$DD = \frac{E(V_1) - DPT}{\sigma}$$

3. Scale the distance to default to actual probabilities of default using a default database

   KMV does not assume a normal distribution when estimating the probability of default (which is called Expected Default Frequency – EDF)

   Rather, KMV uses historical information to estimate the probability of default given the distance to default (DD)

   The calculated EDF does appear to indicate likely defaults; changes in EDFs tend to occur before rating agency downgrades

   The EDF varies quite a bit for firms that have the same rating

   Since rating agencies are slow to change ratings, historical analysis will overstate the true probability of maintaining a particular credit quality

   The historical probability of default in a particular rating class is probably overstated because certain firms should have already been downgraded

**6. KMV's Valuation Model for Cash Flows Subject to Default Risk**

In the CreditMetrics model covered in Chapter 8, the forward value of the bond at the time horizon is just the expected present value of the cash flows after that horizon date

The present value is calculated for each possible rating at the date using the associated spread curve for the discount rate

Each present value is multiplied by its probability of occurring (from the transition matrix) and summed to get the expected present value

The KMV approach is similar to option pricing and uses a risk-neutral valuation model

6.1 Derivation of the "Risk-Neutral" EDFs

As shown in Section 3 of this chapter, the risk neutral probability of default is N(-$d_2$), where $d_2$ is a function of the risk-free rate *r*

To calculate the real world probability of default, μ is substituted for *r* in the equations for $d_1$ and $d_2$ to calculate $d_1^{'}$ and $d_2^{'}$

$$d_2^{'} = \frac{\ln\left(\frac{V_0}{F}\right) + (\mu - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

The risk-neutral probabilities must be derived from the real world EDFs that have been calculated

The formulas in this section get messy quickly

- First solve for $d_2^{'}$ by using $d_2^{'} = N^{-1}(EDF)$
- Then solve for $d_2$ using $d_2^{'} = d_2 + \left(\mu - r\right)\frac{\sqrt{T}}{\sigma}$

## 7. Asset Return Correlation Model

Correlations between the various firms is required to calculate the credit risk for an entire portfolio

This becomes computationally infeasible if there are many firms, because there would be $\frac{N(N-1)}{2}$ required estimates

This can be reduced substantially by using a multifactor model, which assumes there are several common factors that link all the firms

The correlation estimates between each firm are then a function of the firm's variance and the various factor sensitivities, $\beta_{ij}$, for each firm *i* and each factor *j*

KMV constructs a three-layer factor structure

1. Composite Factor

   Split risk into systematic risk and firm-specific risk

   The systematic risk is a weighted sum of the risks determined in step 2
2. Country and Industry Factors
3. Global, Regional, and Sector Factor

## Appendix 1: Integrating Yield Spread with Options Approach

First derive the risk-free and corporate zero-coupon yield curve from the respective current coupon-paying curves

Next calculate the value of a put option that eliminates the credit risk of a corporate bond by using the following equations

$$B_0 = Fe^{-yT} \qquad P_0 = Fe^{-rT} - B_0$$

Then use an iterative approach on the following equation with a particular volatility assumption, σ, to solve for the firm's asset value, $V_0$

$$P_0 = \left[ -\frac{N(-d_1)}{N(-d_2)} V_0 + Fe^{-rT} \right] N(-d_2)$$

The present value of the recovery value of the loan given default is

$$\frac{N(-d_1)}{N(-d_2)} V_0$$

## Appendix 2: Risk-Neutral Valuation Using "Risk-Neutral" EDFs

A single cash flow subject to default risk can be broken into two pieces: default-free component and risky component

Assume Q is the risk-neutral probability of default, LGD is the loss given default percentage, B is the current value of $1 cash flow paid in one year, and R is the risk-free rate for the one-year horizon

$$B = \frac{Q(1-LGD)+(1-Q)}{1+R} = \frac{1-LGD}{1+R} + \frac{LGD(1-Q)}{1+R} = \begin{bmatrix} \text{Risk-free} \\ \text{Cash Flow} \end{bmatrix} + \begin{bmatrix} \text{Risky} \\ \text{Cash Flow} \end{bmatrix}$$

Assuming the value of the bond (B) and the risk-free rate (R) are known, the following equation can be used to solve for the credit spread (CS)

$$B = \frac{1}{1+R+CS} = \frac{1-LGD}{1+R} + \frac{LGD(1-Q)}{1+R}$$

$$\therefore CS = \frac{(LGD)(Q)(1+R)}{1-(LGD)(Q)}$$

The approach is similar with multiple cash flows

**Appendix 3: Limitations of the Merton Model and Some Extensions**

Limitations of the Merton Model

1. The asset value and its volatility are not directly observable
2. The risk-free rate is assumed constant

   Many authors have relaxed this assumption
3. It assumes all debt is a zero coupon debt, so it does not consider the case when default occurs due to missed coupon payments
4. Seniority of debt is not considered

   But seniority often does not mean much in bankruptcy anyway

Implied credit spreads from Merton's model are too low

Extensions of Merton's Model

- Geske extended the model by treating coupons and seniority as compound options
- Black and Cox developed a model that assumes default occurs when some boundary is pierced, even if it is before the bond's maturity
- Logstaff and Schwartz built on the Black and Cox extension by adding a constant recovery rate for each security and a stochastic risk-free interest rate